ON THE SOLUTION OF MOLODENSKIY'S PROBLEM WITH THE AID OF A TAYLOR SERIES

M.I. Marych

Translation of: O reshenii zadachi molodenskogo s pomoshvyu ryada teylora
In: Geodeziya, Kartografiya i Aerofotos'yemka, issue 17, 1973, pp. 26-33.

(NASA-TT-F-15739) ON THE SOLUTION OF MOLODENSKIY'S PROBLEM WITH THE AID OF A TAYLOR SERIES (Linguistic Systems, Inc., Cambridge, Mass.) 16 p HC $4.00 CSCL 12A G3/19

N74-34082

Unclas 49298

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546 SEPTEMBER 1974

STANDARD TITLE PAGE

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TT F-15,739		

4. Title and Subtitle	5. Report Date
ON THE SOLUTION OF MOLODENSKIY'S PROBLEM WITH THE AID OF A TAYLOR SERIES	September, 1974
	6. Performing Organization Code
7. Author(s)	8. Performing Organization Report No.
M.I. Marych	10. Work Unit No.
9. Performing Organization Name and Address	11. Contract or Grant No.
LINGUISTIC SYSTEMS, INC. 116 AUSTIN STREET CAMBRIDGE, MASSACHUSETTS 02139	NASW-2482
	13. Type of Report & Period Covered
12. Sponsoring Agency Name and Address	TRANSLATION
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION WASHINGTON, D.C. 20546	14. Sponsoring Agency Code

15. Supplementary Notes

Translation of: O reshenii zadachi molodenskogo s pomoshvyu ryada teylora In: Geodeziya, Kartografiya i Aerofotos'yemka Issue 17, 1973, pp. 26-33.

(A74-20604)

16. Abstract

The excitation potential at the physical surface of the earth is expanded in a Taylor series in powers of the elevations (heights) of the earth's relief. Formulas are proposed for calculating successive approximations of the excitation potential and the radial components of the earth's gravitational anomaly. It is shown that the problem of the analytical continuation of the gravitational potential into the mass of the earth does not limit the practical use of the method proposed.

17. Key Words (Selected by Author(s))	18. Distribution Statement
	UNCLASSIFIED - UNLIMITED

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
UNCLASSIFIED	UNCLASSIFIED		

Republic Interdepartmental Scientific-Technical Collection
1973 Geodesy, Cartography, and Aerial Photography Issue A

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M. I. Marych

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In the work (2) a new method is examined for solving Molodenskiy's boundary value problem, based on the use of a relation between the Taylor series of anomalies of gravitational force and the excitation potential expanded in powers of elevations of the earth's surface, and of a small parameter established by the Stokes formula. Formulae are developed and investigated, determining the excitation potential at the physical surface of the earth in the zero, first, and second Molodenskiy approximations.

The calculations performed are indicative of the distribution of results obtained in higher order approximations of the excitation potential and in approximations, entering here, of radial derivatives of anomalies of the gravitational force, which are completely analogous to the corresponding excitation potential approximations.

We wish to give a more extensive and systematic presentation of the method, and also, to illuminate it more by demonstration.

1. Since the solution of the problem should not depend on the choice of radius of the reference sphere, then, in deriving a formula, one can assume that the indicated Taylor series are convergent. Actually, however, values of excitation found in this way should not vary with an increase or decrease in the elevations of the earth's relief to a constant magnitude, which eliminates our proposition of series convergence.

This can be interpreted in the following way. The proposition of convergence of the named series means that we consider

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*Numbers in the margin indicate pagination in the foreign text.

some element of the gravitational field pertaining to a point or element mass, arbitrarily expanded within the body of the earth, and the radius of the reference sphere is chosen such that the elevations of the earth's relief occur at a smaller distance from the points than the physical surface of the earth to the given mass. The independence of the solution relative to the magnitude of the radius of the sphere is indicative of the fact that for all elements composing measured anomalies of the gravitational force, only that fixed reference sphere of arbitrarily chosen radius can be assumed. Thus the values of excitation potential found with respect to the measured anomalies of gravitational force will be equal to those values of the excitation potential which are the result of separately performed calculations with respect to all elements of anomalies of gravitational force having reference spheres ensuring convergence of the Taylor series expansions.

And so, we expand the excitation potential on the physical surface of the earth in a Taylor series of powers of elevations of the earth's relief.

$$T_s = T - \frac{\partial T}{\partial \rho} H + \frac{1}{2} \frac{\partial^2 T}{\partial \rho^2} - \dots, \tag{1}$$

where T_s is the value of the excitation potential at points of the reference sphere; the excitation potential T and its radial derivative $\partial T/\partial \rho$, etc., refer to the physical surface of the earth. If this series converges at each point of the earth's physical surface, then it is obvious that /27

$$T_s = \frac{R}{4\pi} \int (\Delta g)_s [s(\psi) - 1] d\sigma. \tag{2}$$

where

$$(\Delta g)_s = \Delta g - \frac{\partial \Delta g}{\partial \rho} H + \frac{1}{2} \frac{\partial^2 \Delta g}{\partial \rho^2} H^2 - \dots \tag{3}$$

In this way, the quantity T_s, being the Taylor series expansion (1) of the excitation potential, can be written in the form of Stokes' formula (2), in which the cover of the integral is a Taylor series expansion of anomalies of gravitational force. Of course, on the basis of the assumption of the convergence of series (1), series (3) is also convergent.

We write equation (2) in the form of a formula for calculating successive approximations of the excitation potential at the earth's physical surface:

$$T = \frac{R}{4\pi} \int \left(\Delta g - \frac{\partial \Delta g}{\partial \rho} H + \frac{1}{2} \frac{\partial^2 \Delta g}{\partial \rho^2} H^2 - \dots \right) [S(\psi) - 1] d\sigma + \\ + \frac{\partial T}{\partial \rho} H - \frac{1}{2} \frac{\partial^2 T}{d\rho^2} H^2 + \dots \tag{4}$$

If the approximations of excitation potential were constructed so that they did not depend on the choice of reference sphere radius and were simultaneously successive approximations of formula (4), then such approximations which are within the limits for an unbounded space of the number of approximations, reduce to the point values of the excitation potential.

The Molodenskiy approximations (3) obtained by means of the expansion of the excitation potential in powers of a small parameter, meet this compulsory requirement of independence of the calculated result from the assumed magnitude of the radius of the reference sphere.

The following calculation process is suitable for the given principle of construction of approximations. Again, we find the excitation potential with inclusion only of the first term Δg of the integrand of the series, and we obtain the null or so-called Stokes approximation T_0, and then T_1, T_2, etc. up to T_0. The first of these is the result of allowance in the formula of the first radial derivatives of anomalies of gravitational force $(\partial\Delta g/\partial\rho)_0$ of the excitation potential $(\partial T/\partial\rho)_0$ found in the Stokes approximation.

The quantity $(\partial\Delta g/\partial\rho)_0$ is the zeroth approximation of the formula for $\partial\Delta g/\partial\rho$, analogous to formula (4). We obtain this formula if, in place of the Taylor series (1) for T, we write this same series for $\partial\Delta g/\partial\rho$ and in place of the Stokes formula (2) we use the well-known Numerov formula for the vertical gradient of an anomaly of gravitational force. We find $(\partial T/\partial\rho)_0$ from the boundary condition for the excitation potential, in which we substitute T_0 in place of T, and R in place of $\rho=R+H$. We calculate the following corrections to T_0 analogously. /28

Thus, the solution of the basic Molodenskiy problem is expressed by successive approximations of a Taylor series (4), in which we have approximations instead of point values of the radial derivatives of anomalies of gravitational force and excitation potential. Moreover, with increase in the order of the derivatives, the order of the approximation is decreased, and derivatives of higher order are expressed only by Stokes approximations.

Therefore, for divergent Taylor series, our approximations are quite different from formula (4). However, bearing in mind the fact that the approximation of excitation potential does not depend on the magnitude of the radius of the sphere, it may always be assumed that the reference sphere goes through the outer core of mass, and subsequently the approximations reduce to a convergent Taylor series in which the higher order terms

are of negligible size. The effect here appears of removing the negative influence of the divergence of the series used in solving Molodenskiy's problem.

In order to show the size of the influence described on the calculated value of excitation potential, we find a modification of formula (4), induced by changing the height H to some constant magnitude ΔH. Let us assume that, for the elevation of the earth's relief, the elevations $H' = H+\Delta H$ ($R'=R-\Delta H$) and let us examine the series

$$\left.\begin{aligned} T'_c &= T - \frac{\partial T}{\partial \rho} H' + \frac{1}{2}\frac{\partial^2 T}{\partial \rho^2} H'^2 - \cdots, \\ T'_c &= \frac{R'}{4\pi}\int \left(\Delta g - \frac{\partial \Delta g}{\partial \rho} H' + \frac{1}{2}\frac{\partial^2 \Delta g}{\partial \rho^2} H'^2 - \cdots\right)[s(\psi) - 1]\, d\sigma. \end{aligned}\right\} \quad (D)$$

We reduce them to the form

$$T'_c = T_c = \left(\frac{\partial T}{\partial \rho}\right)_c \Delta H + \frac{1}{2}\left(\frac{\partial^2 T}{\partial \rho^2}\right)_c \Delta H^2 - \cdots, \quad (5)$$

$$T'_c = T_c - \frac{\Delta H}{R} T_c + \frac{R'}{4\pi}\int \left[-\left(\frac{\partial \Delta g}{\partial \rho}\right)_c \Delta H^2 + \frac{1}{2}\left(\frac{\partial^2 \Delta g}{\partial \rho^2}\right)_c \Delta H^2 - \cdots\right][s(\psi) - 1]\, d\sigma. \quad (5')$$

Here

$$\left.\begin{aligned} \left(\frac{\partial^k T}{\partial \rho^k}\right)_c &= \frac{\partial^k T}{\partial \rho^k} = \sum_{n=k+1}^{\infty} \frac{(-1)^{n-1}}{(n-k)!} \frac{\partial^n T}{\partial \rho^n} H^{n-k}, \\ \left(\frac{\partial^k \Delta g}{\partial \rho^k}\right)_c &= \frac{\partial^k \Delta g}{\partial \rho^k} + \sum_{n=k+1}^{\infty} \frac{(-1)^{n-1}}{(n-k)!} \frac{\partial^n \Delta g}{\partial \rho^n} H^{n-k} \end{aligned}\right\} \quad (E)$$

the sums (k=1, 2, ...) are Taylor series expansions in powers of the elevations H of the radial derivatives of excitation potential and anomalies of the gravitational force. Differentiating the Stokes formula (2) along the direction of the radius vector ρ, we find /29

$$\left(\frac{\partial^k \Delta T}{\partial \rho^k}\right)_c = \frac{k}{4\pi}\int\left(\frac{\partial^{k-1}\Delta g}{\partial \rho^{k-1}}\right)_c [s(\psi)-1]\,d\sigma + \frac{R}{4\pi}\int\left(\frac{\partial^k \Delta g}{\partial \rho^k}\right)_c [s(\psi)-1]\,d\sigma \tag{6}$$

Inverting these expressions into (5), and (2) into the second term of formula (5'), combining the series (5) and (5') with each other we obtain

$$T_c' - T_c'' = \delta_k,$$

where

$$\delta_k = \frac{(-1)^k \Delta H^{k+1}}{4\pi k!}\int\left(\frac{\partial^k \Delta g}{\partial \rho^k}\right)_c [s(\psi)-1]\,d\sigma \approx \frac{(-\Delta H)^k}{k!}\frac{\partial^k T}{\partial \rho^k}\frac{\Delta H}{R} \tag{7}$$

is the desired modification of the excitation potential induced by substituting the elevations H' for the elevations H in formula (4), in which are estimated radial derivatives up to the kth order inclusively. We note that the given expression corresponds completely to the result obtained in (1) without consideration of the earth's relief.

Thus, if the indicated substitution of elevations does not lead to divergence of series (1) and (3) at the investigated point, then the quantity δk with an increase in the number k is established as sufficiently small. If also at the given point the indicated series diverge, then with an increase in k, the quantity δk increases and is established to be sufficiently large.

In calculating excitation potential according to formula (4), when the necessity arises of breaking off the series, and we have only to deal with some quantity of their first terms. In the case that these series converge at all points of the earth's physical surface, cutting them off does not cause ($\delta_k \to 0$) and in the process of calculation, a sufficiently precise result can be obtained. However, for an arbitary distribution from inside the earth the difference between found and desired results can be unboundedly large ($\delta_k \to \infty$). For the earth's reality, the indicated requirement is too strong, and is not fulfilled; this is the chief obstacle to using a Taylor series for calculating excitation potential. In the given method for determining excitation potential, the indicated obstacle is removed by the use of Molodenskiy's parameter.

Independently of us (1, 2), the German scholar H. Morits examined an analogous method of solving this same problem, based also on the use of a Taylor series and the small parameter k (5). With the aid of asymptotic series and expansions of the excitation potential in powers of the parameter k, he shows that Molodenskiy's formulas and Brovar's formulas are equivalent to his formulas. This agrees completely with us and the conclusions (1, 2). However, H. Morits does not consider the above-indicated role of the parameter k in constructing his formulas and therefore, he apparently does not discover that they do not depend on the possibility of analytic continuation of the gravitational potential inside the earth's mass, accomplished with the aid of a Taylor series.

It is important to note that in work (6), H. Morits arrived at conclusions which essentially coincide with our conclusions (1, 2).

2. We obtain formulas for calculating the excitation potential, which follow from the Taylor series expansion of this potential (4) in powers of the parameter k ($0 \leq k \leq 1$).

$$T_0+kT_1+k^2T_2+\ldots=\frac{R}{4\pi}\int\left\{\Delta g-\left[\left(\frac{\partial\Delta g}{\partial\rho}\right)_0+k\left(\frac{\partial\Delta g}{\partial\rho}\right)_1+\ldots\right]kH+\right.$$
$$+\frac{1}{2}\left[\left(\frac{\partial^2\Delta g}{\partial\rho^2}\right)_0+\ldots\right]k^2H^2-\ldots\Big\}[s(\psi)-1]\,d\sigma+$$
$$+\left[\left(\frac{\partial T}{\partial\rho}\right)_0+k\left(\frac{\partial T}{\partial\rho}\right)_1+\ldots\right]kH-\frac{1}{2}\left[\left(\frac{\partial^2 T}{\partial\rho^2}\right)_0+\ldots\right]k^2H^2+\ldots \tag{8}$$

Equating terms of both parts of the equation with themselves containing the same power of k, we have

$$T_0=\frac{R}{4\pi}\int\Delta g\,[s(\psi)-1]\,d\sigma; \tag{9}$$

$$T_1=\frac{R}{4\pi}\int\left[-\left(\frac{\partial\Delta g}{\partial\Delta}\right)_0 H\right][s(\psi)-1]\,d\sigma+\left(\frac{\partial T}{\partial\rho}\right)_0 H; \tag{10}$$

$$T_2=\frac{R}{4\pi}\int\left[-\left(\frac{\partial\Delta g}{\partial\rho}\right)_1 H+\frac{1}{2}\left(\frac{\partial^2\Delta g}{\partial\rho^2}\right)_0 H^2\right][s(\psi)-1]\,d\sigma+\left(\frac{\partial T}{\partial\rho}\right)_1 H-\frac{1}{2}\left(\frac{\partial^2 T}{\partial\rho^2}\right)_0 H^2; \tag{11}$$

$$T_n=\frac{R}{4\pi}\int\sum_{m=1}^{n}\frac{(-1)^m}{m!}\left(\frac{\partial^m\Delta g}{\partial\rho^m}\right)_{n-m}H^m[s(\psi)-1]\,d\sigma+$$
$$+\sum_{m=1}^{n}\frac{(-1)^{m-1}}{m!}\left(\frac{\partial^m T}{\partial\rho^m}\right)_{n-m}H^m. \tag{12}$$

The sum of all the terms of the given expansion

$$T=\sum_{n=0}^{\infty}T_n \tag{13}$$

gives the excitation potential on the physical surface of the earth.

And so, we have obtained formulas for calculating successive approximations of the excitation potential. The first of these (9) coincides with the zeroth Molodenskiy approximation (3). This is the ordinary Stokes formula (Stokes approximation); it is suitable, as is evident from its derivation, for calculating excitation potential on the earth's physical surface without allowing for the relief.

In order to obtain formulas for successive approximations of radial derivatives of anomalies of gravitational force, we make use of a Taylor series and the relations (1)

$$\left(\frac{\partial^i \Delta g}{\partial \rho^i}\right)_s = \frac{1}{2\pi R}\int\left[\left(\frac{\partial^{i-1}\Delta g}{\partial \rho^{i-1}}\right)_s - \left(\frac{\partial^{i-1}\Delta g}{\partial \rho^{i-1}}\right)_s^{(0)}\right]\frac{d\sigma}{r^3} - \frac{i+1}{R}\left(\frac{\partial^{i-1}\Delta g}{\partial \rho^{i-1}}\right)_s, \tag{14}$$

(i = 1, 2, ...) in precisely the same way as we used the Stokes formula for finding excitation potential. Expanding the first derivative $\partial\Delta g/\partial\rho$ in a Taylor series in powers of elevations of the earth's relief

$$\left(\frac{\partial \Delta g}{\partial \rho}\right)_s = \frac{\partial \Delta g}{\partial \rho} - \frac{\partial^2 \Delta g}{\partial \rho^2} H + \frac{1}{2}\frac{\partial^3 \Delta g}{\partial \rho^3} H^2 - \ldots \tag{F}$$

and bearing in mind the fact that

$$\left(\frac{\partial \Delta g}{\partial \rho}\right)_s = \frac{1}{2\pi R}\int\left[(\Delta g)_s - (\Delta g)_s^{(0)}\right]\frac{d\sigma}{r^3} - \frac{2(\Delta g)_s}{R}, \tag{G}$$

$(\Delta g)_s$ is the Taylor series expansion (3) of an anomaly of the gravitational force. We obtain

$$\frac{\partial \Delta g}{\partial \rho} = \frac{1}{2\pi R}\int [(\Delta g)_\sigma - (\Delta g)_\sigma^{(0)}]\frac{d\sigma}{r^3} - \frac{2(\Delta g)_\sigma}{R} + \frac{\partial^2 \Delta g}{\partial \rho^2} H - \frac{1}{2}\frac{\partial^3 \Delta g}{\partial \rho^3} H^2 + \ldots \qquad \text{(H)}$$

This formula is analogous to formula (4). Presenting it in the form of an expansion in powers of the parameter k ($0 \leq k \leq 1$), we find relations analogous to (8). Further, equating to each other terms containing the same power of k, we obtain formulas determining successive approximations of the values of the first radial derivative of the anomaly of gravitational force at points of the earth's physical surface. And so, we have

$$\left(\frac{\partial \Delta g}{\partial \rho}\right)_0 = \frac{1}{2\pi R}\int (\Delta g - \Delta g^{(0)})\frac{d\sigma}{r^3} - \frac{2}{R}\Delta g; \qquad (15)$$

$$\left(\frac{\partial \Delta g}{\partial \rho}\right)_1 = \frac{1}{2\pi R}\int \left\{-\left[\left(\frac{\partial \Delta g}{\partial \rho}\right)_0 H - \left(\frac{\partial \Delta g}{\partial \rho}\right)_0^{(0)} H_0\right]\right\}\frac{d\sigma}{r^3} - \\ - \frac{2}{R}\left[-\left(\frac{\partial \Delta g}{\partial \rho}\right)_0 H\right] + \left(\frac{\partial^2 \Delta g}{\partial \rho^2}\right)_0 H; \qquad (16)$$

$$\left(\frac{\partial \Delta g}{\partial \rho}\right)_n = \frac{1}{2\pi R}\int \sum_{m=1}^{n} \frac{(-1)^m}{m!}\left[\left(\frac{\partial^m \Delta g}{\partial \rho^m}\right)_{n-m} H^m - \left(\frac{\partial^m \Delta g}{\partial \rho^m}\right)_{n-m}^{(0)} H_0^m\right]\frac{d\sigma}{r^3} - \\ - \frac{2}{R}\sum_{m=1}^{n} \frac{(-1)^m}{m!}\left(\frac{\partial^m \Delta g}{\partial \rho^m}\right)_{n-m} H^m + \sum_{m=1}^{n} \frac{(-1)^{m-1}}{m!}\left(\frac{\partial^{m+1} \Delta g}{\partial \rho^{m+1}}\right)_{n-m} H^m. \qquad (17)$$

subsequently

$$\frac{\partial \Delta g}{\partial \rho} = \sum_{n=0}^{\infty}\left(\frac{\partial \Delta g}{\partial \rho}\right)_n. \qquad (18)$$

The derivation of formulas for approximating the second and higher order derivatives does not differ from the derivation of formulas (15)-(18). Therefore, not repeating the calculations based on the use of relations (14) and of the Taylor series, one can immediately write the expressions

$$\left(\frac{\partial^i \Delta g}{\partial \rho^i}\right)_0 = \frac{1}{2\pi R}\int\left[\left(\frac{\partial^{i-1}\Delta g}{\partial \rho^{i-1}}\right)_0 - \left(\frac{\partial^{i-1}\Delta g}{\partial \rho^{i-1}}\right)_0^{(0)}\right]\frac{d\sigma}{r^3} - \frac{i+1}{R}\left(\frac{\partial^{i-1}\Delta g}{\partial \rho^{i-1}}\right)_0, \tag{19}$$

$$\left(\frac{\partial^i \Delta g}{\partial \rho^i}\right)_n = \frac{1}{2\pi R}\int\sum_{m=1}^{n}\frac{(-1)^m}{m!}\left[\left(\frac{\partial^{i+m-1}\Delta g}{\partial \rho^{i+m-1}}\right)_{n-m} H^m - \left(\frac{\partial^{i+m-1}\Delta g}{\partial \rho^{i+m-1}}\right)_{n-m}^{(0)} H_0^m\right]\frac{d\sigma}{r^3} - \frac{2}{R}\sum_{m=1}^{n}\frac{(-1)^m}{m!}\left(\frac{\partial^{i+m-1}\Delta g}{\partial \rho^{i+m-1}}\right)_{n-m} H^m + \sum_{m=1}^{n}\frac{(-1)^{m-1}}{m!}\left(\frac{\partial^{i+m}\Delta g}{\partial \rho^{i+m}}\right)_{n-m} H^m \tag{20}$$

(i=1,2,3,...), determining

$$\frac{\partial^i \Delta g}{\partial \rho^i} = \sum_{n=0}^{\infty}\left(\frac{\partial^i \Delta g}{\partial \rho^i}\right)_n \tag{21}$$

(i=1,2,3,...).

In such a manner, expressions (15)-(18) and (19)-(21), obtained in the same way as expressions (9)-(13), are completely analogous to them. The process for calculating the first radial derivative of the anomaly of gravitational force (18) and derivatives of higher order (21) is the same as the process for calculating excitation potential. For this case, the zeroth (Stokes) approximation of derivatives of higher orders, entering into formulas (16)-(18) and into (20)-(21) (denoted by the lower index "0"), and the subsequent corrections to these approxima-

tions are determined by formulas obtained from (20) for suitable values of i and n.

We note that, in practice, when using formulas (15)-(17) and (19)-(20), their second terms can be neglected. In this case, as shown in work (2), the Ostach formula (4) for calculating the first Molodenskiy correction to the Stokes approximation of the vertical gradient of the anomaly of gravitational force (15) (to the Numerov formula) is easily reduced to formula (16).

We examine also the process of finding approximations for radial derivatives of the excitation potential.

Differentiating the boundary condition for excitation potential

$$\frac{\partial T}{\partial \rho} + \frac{2T}{\rho} = -\Delta g \tag{22}$$

along the direction of the radius vector ρ, we find subsequently

$$\left.\begin{aligned} \frac{\partial^2 T}{\partial \rho^2} &= -\frac{\partial \Delta g}{\partial \rho} + \frac{2}{\rho}\Delta g + \frac{6}{\rho^2}T, \\ \frac{\partial^3 T}{\partial \rho^3} &= -\frac{\partial^2 \Delta g}{\partial \rho^2} + \frac{2}{\rho}\frac{\partial \Delta g}{\partial \rho} - \frac{8}{\rho^2}\Delta g - \frac{24}{\rho^3}T \end{aligned}\right\} \tag{23}$$

etc.

Representing relation (22), in which

$$\frac{1}{\rho} = \frac{1}{R}\left(1 - \frac{H}{R} + \frac{H^2}{R^2} - \dots\right), \tag{J}$$

in the form of an expansion in powers of the parameter k and equating to each other the terms of both parts of the equality

containing the same power of k, we obtain

$$\left(\frac{\partial T}{\partial \rho}\right)_0 = -\Delta g - \frac{2T_0}{R}; \qquad \left(\frac{\partial T}{\partial \rho}\right)_1 = -\frac{2}{R}\left(T_1 - \frac{H}{R}T_0\right);$$
$$\left(\frac{\partial T}{\partial \rho}\right)_2 = -\frac{2}{R}\left(T_2 - \frac{H}{R}T_1 + \frac{H^2}{R^2}T_0\right) \tag{K}$$

etc.

In this same way we also find an approximation of the derivatives (23). We have

$$\left(\frac{\partial^2 T}{\partial \rho^2}\right)_0 = -\left(\frac{\partial \Delta g}{\partial \rho}\right)_0 + \frac{2}{R}\Delta g + \frac{6}{R^2}T_0;$$
$$\left(\frac{\partial^2 T}{\partial \rho^2}\right)_1 = -\left(\frac{\partial \Delta g}{\partial \rho}\right)_1 - \frac{2H}{R^2}\Delta g + \frac{6}{R^2}T_1 - \frac{12H}{R^3}T_0;$$
$$\left(\frac{\partial^3 T}{\partial \rho^3}\right)_0 = -\left(\frac{\partial^2 \Delta g}{\partial \rho^2}\right)_0 + \frac{2}{R}\left(\frac{\partial \Delta g}{\partial \rho}\right)_0 - \frac{8}{R^2}\Delta g - \frac{24}{R}T_0 \tag{L}$$

etc.

Investigations of formulas (9)-(13), determining successive approximations of the excitation potential, identical to the investigation of the Taylor series (4), connected with obtaining the quantity δ_k (7), shows (cf. (2)) that the result of the calculation does not depend on decrease or increase of the radius of the reference sphere, that is, in the case presented, $\delta_k = 0$, an increase of the elevations of the earth's relief ($H' = H+\Delta H$, $R^1 = R-\Delta H$), even if it leads to divergence of series used for derivation of formulas for successive approximations of excitation potential at the physical surface of the earth (the reference sphere, which is the anomalous

mass presented above, is now below it); the result of the calculation remains unchanged.

In this way, it is proven that the question of convergence of the Taylor series expansion of the anomaly of gravitational force in powers of elevations of the earth's relief, and also the question of analytic continuation of the gravitational potential, do not determine the applications of the method examined.

Work delivered to the editorial board May 10, 1972. Recommended to the Department of Higher Geodesy and Gravimetry of the L'vovsky order of Lenin Poly-Technic Institute.